I. **Comment by DAVID EUGENE SMITH, Ph. D., Professor of Mathematics, Michigan State Normal School, Ypsilanti, Mich.**

This question seems to me to have no interest to mathematicians. It simply means that somebody has set up a narrow definition of multiplication and has then said, "*Your* work is not multiplication because it does not fit *my* definition." I expressed my humble opinion in the MONTHLY some time ago when the antiquated definition of division was brought up to prove that it was impossible to divide \$10 by 2. Such narrow limitations seem to me utterly nonsensical.

In a similar sense we cannot multiply by -1, and we cannot have "imaginary numbers," and 1 is not a number, etc., etc. Mathematical progress has always been made the more difficult because somebody has insisted on hanging on to some antiquated definition.

What do these people who say that we cannot multiply 2 by 3 say to some such simple formula as $e^{\pi i}=-1$? I suppose they say that e, π, i, and -1 have no existence.

II. **Comment by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.**

There may be a modern idea that since neither 6 nor 4 is a numbered quantity, the operation is impossible. If it were possible to get the evidence of all the mathematicians, I am sure not one could be found who did not learn his multiplication table, in fact, get his aptness in numbers by the same process as given in the problem. There may be some who claim otherwise but I would even doubt their claim.

$6\times4=24$ is good arithmetic. It seems a pity that vandals should make incursions upon the sacred shrines of Newton, La Place, Pierce, and other noted men of numbers, and so desecrate their immortal works, as to try and mistify their teachings. The God of Mathematics will not permit it.

III. **Remarks by J. K. ELLWOOD, A. M., Principal of the Colfax School, Pittsburg, Pa.**

Do $4\times6=24$? Can 6 be multiplied by 4 ? Six what ? If 6 units of quantity, yes; but if not a magnitude,—well, what then is it ? "Six" apart from the universe of space, time, and matter, suggests to the mind—what ? The "how many"? The *ratio* of the "how many" to the unit ? Six in the abstract —a pure number—can not, in an arithmetical sense, be multiplied by any abstraction. In an algebraic sense, $4\times6=24$, just as $x\times x=x^2$. That is, we operate with *symbols*, neglecting the realities represented. If two abstract numbers can be multiplied one by the other, why not two concrete numbers, as feet$\times$feet =square feet ?

42. **Proposed by E. B. ESCOTT, Cambridge, Mass.**

To find triangles whose sides and median lines are commensurable.

II. **Solutions communicated to "L' Intermediaire des Mathematiciens" (January, 1898) by the PROPOSER. Selected and translated by J. M. COLAW, A. M., Monterey, Va.**

First Solution. By Chas. Gill (New York, 1848).

$x=t[1-(\cos A+\sin A)(\cos B+\sin B)]$, $y=t[\cos B-\sin B+(\cos A-\sin A)(\cos B+\sin B)]$, $z=t[\cos A-\sin A+(\cos B-\sin B)(\cos A+\sin A)$, whence there exists one of the four relations following:

$\tan\frac{1}{2}A=(16+13\sin B-8\cos B-5\sin 2B-2\sin^2 B)/[(2+\sin B+2\cos B)(5+4\sin B-4\cos B)]\ldots\ldots$(I) ; $\tan\frac{1}{2}A=[(1-\sin B)(5-\cos B+4\sin B)]/[(\sin B+\cos B)(1+3\cos B-\sin B)]\ldots\ldots$(II) ; $\cot\frac{1}{2}A=-(16-13\cos B+8\sin B-5\sin 2B-2\cos^2 B)/[(2-\cos B-2\sin B)(5+4\sin B-4\cos B)\ldots\ldots$(III) ; $\cot\frac{1}{2}A=[(1+\cos B)(5+\sin B-4\cos B)]/[(\sin B+\cos B)(1+\cos B-3\sin B)]\ldots\ldots$(IV). Letting, in formula (II), $\sin B=\frac{4}{5}$, $\cos B=-\frac{3}{5}$, then $\tan\frac{1}{2}A=-\frac{11}{2}$, $x=262$, $y=316$, $z=254$, $m_x=261$, $m_y=204$, and $m_z=255$.

Second Solution. From *The Gentleman's Mathematical Companion*, London, 1824, page 350.

Let $z=x+y-d$, then $4m_z^2=x^2-2xy+y^2+2d(x+y)-d^2$; $4m_y^2=4x^2+4xy+y^2-4d(x+y)+2d^2$; $4m_x^2=x^2+4xy+4y^2-4d(x+y)+2d^2$.

Let $4x^2+4xy+y^2-4d(x+y)+2d^2=(2x+y-m)^2$; then $x=(2d^2-4dy+2my-m^2)/(4d-4m)$, $y=(2d^2-4dx+4mx-m^2)/(4d-2m)$.

Let $x^2+4xy+4y^2-4d(x+y)+2d^2=(x+2y-n)^2$; then $x=(2d^2-4dy+2ny-n^2)/(4d-2n)$; $y=(2d^2-4dx+2nx-n^2)/(4d-4n)$.

$x=[d^2(4n-2m)+2d(m^2-n^2)-mn(2m-n)]/[4d(m+n)-6mn]$;
$y=[d^2(4m-2n)-2d(m^2-n^2)-mn)2n-m)]/[4d(m+n)-6mn]$;
$z=x+y-d=-[2d^2(m+n)-6mnd+mn(m+n)]/[4d(m+n)-6mn]$.

We may neglect the common denominator $4d(m+n)-6mn$. We then have to satisfy the condition, $2x^2+2y^2-z^2=36d^4(m-n)^2-24d^3(m+n)(2m^2-5mn+2n^2)+4d^2(4m^4+7m^2n-39m^2n^2+7mn^3+4n^4)-12dmn(m+n)(2m^2-5mn+2n^2)+9m^2n^2(m-n)^2=$a square.

We have also $=\{6d^2(m-n)-2d[(m+n)/(m-n)](2m^2-5mn+2n^2)-3mn(m-n)\}^2$.

Whence $d=[3(m+n)(m-n)^2]/(5m^2-8mn+5n^2)$.

Letting $m=3$, $n=2$, $d=\frac{15}{17}$, $x=656$, $y=414$, $z=290$,
$m_x=142$, $m_y=463$, $m_z=529$,
$m=3$, $n=1$, $d=\frac{24}{13}$, $x=174$, $y=170$, $z=136$,
$m_x=127$, $m_y=131$, $m_z=158$,
$m=3$, $n=-1$, $d=\frac{48}{37}$, $x=650$, $y=318$, $z=628$,
$m_x=377$, $m_y=619$, $m_z=404$,
$m=5$, $n=4$, $d=\frac{3}{5}$, $x=892$, $y=554$, $z=954$,
$m_x=640$, $m_y=881$, $m_z=569$.

[*E. Fauquembergue* says, (*L' Intermediaire*, *Mars* 1897), that Euler, at different times busied himself with the problem of finding a triangle whose sides and medians are commensurable. His solution is reproduced in the *Commentationes Arithmeticæ collectæ*, *t.* II, page 488. He gives formulæ from which may be obtained an indefinite number of triangles answering the conditions of the problem. He deduces, among others, the solution with the integers 68, 85, 87 for the sides. See pages 94-95, of MONTHLY, Vol. IV, 1897, for solution I, by M. Tesch. EDITOR.]